The botanist keeps tasting tea

A gentle introduction to e-values and sequential statistical inference

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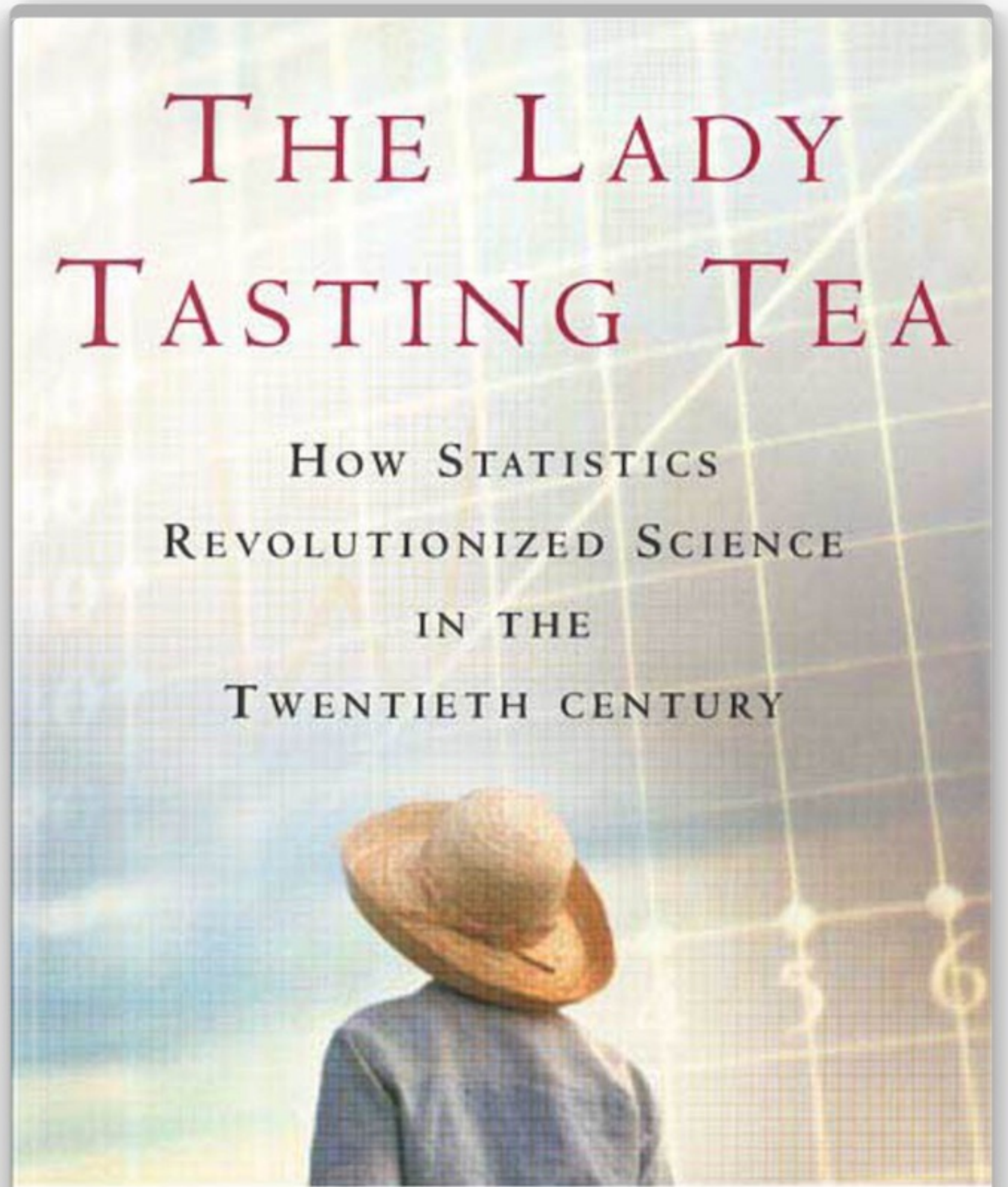

DAVID SALSBURG

"A fascinating description of the kinds of people who interacted, collaborated, disagreed, and were brilliant in the development of statistics."
—Barbara A. Bailar, National Opinion research Center

The botanist tasting tea (1920s)

Ronald Fisher

Muriel Bristol

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What's the probability that a *chance guess* would be *perfect*?

$1/70 \approx 0.014$

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This is a p-value for H_0 : Muriel cannot distinguish btwn MT and TM.

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That is, the type-I error is not controlled: $\mathbb{P}_{H_0}(P_\tau \leq 0.05) \nleq 0.05.$

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An *e-value* is a function E of the data (MT or TM) so that

$$E \geq 0 \quad \text{and} \quad \mathbb{E}_{H_0}(E) \leq 1.$$

Suppose Muriel keeps tasting tea:

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Then, $\mathbb{P}_{H_0}\left(P^\star_\tau \leq 0.05\right) \leq 0.05$, at any data-dependent sample size τ!

where $P^\star_n := \left(E_1 \cdot E_2 \ \cdots \ E_n\right)^{-1}$.

Thank you!

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